Anomalous scaling in two and three dimensions for a passive vector field advected by a turbulent flow

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Abstract

A model of the passive vector field advected by the uncorrelated in time Gaussian velocity with power-like covariance is studied by means of the renormalization group and the operator product expansion. The structure functions of the admixture demonstrate essential power-like dependence on the external scale in the inertial range (the case of an anomalous scaling). The method of finding independent tensor invariants in the cases of two and three dimensions is proposed to eliminate linear dependencies between the operators entering into the operator product expansions of the structure functions. The constructed operator bases, which include the powers of the dissipation operator and the enstrophy operator, provide the possibility of calculating the exponents of the anomalous scaling.

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1. Introduction

The term 'anomalous scaling' refers to deviations from the predictions of the Kolmogorov theory. Such deviations take place, in particular, in the behaviour of the structure functions $S_n(r)$ of the turbulent velocity field **v** in the inertial range $l \ll r \ll L$, where L and l are the external scale and the dissipation length. The structure functions of the vector field φ are defined as

$$S_n(r) \equiv \langle [\varphi_r(t, \mathbf{x} + \mathbf{r}) - \varphi_r(t, \mathbf{x})]^n \rangle, \qquad \varphi_r \equiv \varphi_i r_i / r, \tag{1}$$

For the field **v** the Kolmogorov theory predicts $S_n(r) \sim r^{n/3}$, while the experiments indicate that $S_n(r) \sim r^{n/3 - \xi_n}$ with certain nontrivial 'anomalous exponents' ξ_n.

The Kolmogorov theory states that the only dimensional parameter which determines the statistics of the turbulent velocity pulsations is the average rate of energy dissipation per unit mass $\bar{\varepsilon} = \nu \langle \Phi_{\text{dis}} \rangle$, where ν is the kinematic viscosity and $\Phi_{\text{dis}} \equiv (\partial_i v_j + \partial_j v_i)^2 / 2$

is the operator of local energy dissipation. In stationary state, it is held that $\bar{\varepsilon} = W$ with the power of energy pumping W. In this case the Kolmogorov hypothesis can be formulated for the structure functions (1) as follows. Dimensional analysis leads to the relation $S_n(r, \nu, W, L) = (Wr)^{n/3} R_n(r/l, r/L)$, where $l \equiv (\nu^3/W)^{1/4}$. Then the Kolmogorov hypothesis states that the function R_n has a finite limit in the inertial interval: $R_n(\infty, 0) = \text{const} \neq 0$.

At present the possibility of taking limit in the $R_n(r/l, r/L)$ on the first argument is universally recognized, i.e. the quantity $R_n(r/L) \equiv R_n(\infty, r/L)$ is recognized finite. Hence the structure functions are considered to be independent on the viscosity in the IR region $r \gg l$. If the Kolmogorov hypothesis about finiteness of the $R_n(r/L)$ in the limit $r/L \to 0$ is violated and the asymptotic behaviour is power-like $R_n(r/L) \sim (r/L)^{-\xi_n}$, then the anomalous scaling appears. In phenomenological generalizations of the Kolmogorov theory the anomalous scaling is usually considered as a result of fluctuations of the energy dissipation rate [1, 2].

The analogy with the theory of critical phenomena suggests itself when the problem is formulated as the field-theoretical model. Then the direct analogy to the theory of critical phenomena provides the possibility of applying the powerful UV-renormalization technique to the problem. In this way the first Kolmogorov hypothesis can be proved (independence of the structure functions on the viscosity in the inertial range). The behaviour of $R_n(r/L)$ in the range $r/L \ll 1$ is not determined by the renormalization group itself, the method to find it is the operator product expansion (OPE) which gives the asymptotic expansion

$$S_n(r) \propto r^{n/3} \sum_F (r/L)^{\Delta_F} A_F, \tag{2}$$

where the summation runs over all possible scalar operator products F (constructed from local products of the fields $v_i(t, \mathbf{x})$ and their derivatives), Δ_F are the critical dimensions of the operators [3]. More precisely, Δ_F are the eigenvalues of the matrix of critical dimensions, and summation in (2) runs over the eigenvectors of the matrix. The operators entering into the OPE are those which appear in the Taylor expansion and all the operators that admix to them in renormalization.

If for each F holds $\Delta_F > 0$ (as in the theory of phase transitions) then the terms in (2) determine corrections to the Kolmogorov scaling. If there is an operator in (2) with $\Delta_F < 0$—'dangerous operator' then the limit $r/L \to 0$ in (2) does not exist which leads to the anomalous scaling. Realization of the described scenario meets however two difficulties: technical difficulty of calculation of the critical dimensions Δ_F and fundamental one—if in the theory a dangerous operator exists then it can be shown that with necessity there are infinitely many dangerous operators, the spectrum of their critical dimensions is unbounded below (we cannot point at 'the most dangerous' operator). Thus the expansion (2) can be useful when summation of the series is possible, or when (2) contains only finite number of terms due to the model features.

2. A passive scalar admixture

Recently, significant progress in the description of the anomalous scaling has been achieved in related problems of the turbulent advection of a passive admixture. The experiments and computer simulation data demonstrate that the anomalous scaling appears not only in the turbulent pulsations of the velocity, but much more it reveals in the properties of the field transferred by the turbulent flow $\theta(t, \mathbf{x})$ which may be the field of the admixture concentration or the temperature field. The passive scalar advected by the turbulent velocity field $v_i(t, \mathbf{x})$ is

described by the equation

$$\partial_t \theta + (v_j \partial_j)\theta = \nu_0 \partial^2 \theta + f, \tag{3}$$

where ν_0 is the diffusivity (or the thermal diffusivity), f is the source of the field θ. Significant progress in the description of the anomalous scaling has been achieved in a relatively simple model, due to Kraichnan [4, 5], of a passive scalar advection which assumes the velocity field to be delta-correlated in time and Gaussian with covariance

$$\langle v_i(t+\tau, \mathbf{x}+\mathbf{r})v_j(t, \mathbf{x})\rangle = D_0 \delta(\tau) \int \frac{\mathrm{d}\mathbf{k}}{(2\pi)^d} P_{ij}(\mathbf{k})N(k)\exp\mathrm{i}\mathbf{k}\mathbf{r}, \tag{4}$$

where $P_{ij}(\mathbf{k}) \equiv \delta_{ij} - k_i k_j/k^2$ is the transverse projector (the consequence of transversality of the velocity), the function $N(k)$ in d-dimensional space is modelled with the power-like expression $N(k) \equiv k^{-d-\varepsilon}$ and is supposed to be somehow IR regularized on the scale $k \sim L^{-1}$. Here $0 \leqslant \varepsilon \leqslant 2$ is a kind of Hölder exponent which measures the 'roughness' of the velocity field. In the RG approach, it plays the same role as the parameter $4-d$ in the theory of critical phenomena. The physical value of parameter ε is $\varepsilon = 4/3$, for which the amplitude factor D_0 has the same dimension as the energy pumping power per unit mass.

In the model (3)–(4), the existence of the anomalous scaling was substantiated and the exponents ξ_n of the structure functions were calculated [6–9]. The first results were obtained using the Hopf equations on the distribution function of the equal time fluctuations of the admixture field. The linearity of equation (3) for θ results in closed equations (instead of chain of equations) for the equal time correlation functions. The analysis of so-called 'zero modes' allows one to substantiate the anomalous scaling and to calculate the anomalous exponents. The pair function is founded exactly and has no anomalous scaling ($\xi_2 = 0$). The exponents of the high-order functions were calculated approximately with small parameters ε [6, 7] and $1/d$ [8, 9]:

$$\xi_n = n(n-2)\varepsilon/2(d+2) + O(\varepsilon^2), \tag{5}$$

$$\xi_n = n(n-2)\varepsilon/2d + O(1/d^2). \tag{6}$$

The analysis of the zero modes gives also interesting results in another case—in the Batchelor limit $\varepsilon = 2$ [10–13].

Until now the technical difficulties do not allow us to calculate higher-order terms of the expansions (5) using the analysis of the zero modes. The methods of RG and OPE turns out to be more effective to find the asymptotic approximation on small ε. The terms $\sim \varepsilon^2$ [3] and $\sim\varepsilon^3$ [14] were calculated using these methods. It is essential that in the Kraichnan model the finite number of terms contribute to the OPE (2) of the structure function S_n, precisely, those are the powers of the dissipation operator of the admixture field Φ_{dis}^k with $1 \leqslant k \leqslant n/2$. Joining of asymptotic expansion of the exponents ξ_n on small ε (taking three terms of the expansion) with asymptotic approximation near the Batchelor limit($\varepsilon = 2$) results in interpolation for $\xi_n(\varepsilon)$ which coincides with the data of numerical experiment (within the experimental errors) in the range $0 \leqslant \varepsilon \leqslant 2$, including the physical value $\varepsilon = 4/3$ [14].

The RG method demonstrates the universality of the approach besides technical efficiency. Using the RG the anomalous exponents of order ε^2 in the model of compressible fluid were calculated [15], the model of advection by the Gaussian velocity field with finite correlation time was analysed in [16, 17], the Kazantsev model of the magnetic hydrodynamics—in [15].

3. A passive vector admixture

In the present paper we concentrate on the model of advection of a passive vector admixture proposed in [18–21]. The model is the direct generalization of (3) and is described by the equation

$$\partial_t \varphi_i + (v_j \partial_j)\varphi_i = \nu_0 \partial^2 \varphi_i - \partial_i P + f_i, \tag{7}$$

which come out from equation (3) after replacing the scalar field θ with the transversal vector field φ_i and adding the pressure gradient P to the left-hand side to ensure transversality $\partial_i \varphi_i = 0$. The random force f_i in (7) obeys the Gaussian distribution with zero mean and the covariance

$$\langle f_i(t + \tau, \mathbf{x} + \mathbf{r}) f_j(t, \mathbf{x}) \rangle = \delta(\tau) C_{ij}(\mathbf{r}/L). \tag{8}$$

The exact form of the function $C_{ij}(\mathbf{r}/L)$ is not important. It only models the energy influx resulting from the interaction with large vortices, which compensates the dissipation losses.

Note that all the odd-order structure functions vanish because equation (7) is invariant with respect to the substitution $\varphi \to -\varphi$, $f \to -f$ and the fact that f is Gaussian (the pressure term can be eliminated from (7) by applying the transverse projector).

The model (4), (7), (8) can be considered as a rough approximation describing the turbulent velocity field, when φ is regarded as the 'hard' component of the velocity field and v is regarded as the 'soft' one. Then in the Navier–Stokes (NS) equation for the hard component only the convective transfer by the soft component is taken into account and the soft component obeys the Gaussian distribution with covariance (4). These assumptions do not violate Galilean invariance of the model. Since the structure functions (1) are Galilean invariant, it follows that OPE (2) contains only contributions from the operators which are also Galilean invariant, just as in the stochastic hydrodynamics which is described by the NS equation.

The model (4), (7), (8) was analysed by means of the Hopf equation for the pair correlation function [19]. The function turns out to be non-anomalous ($\xi_2 = 0$) in the isotropic case. The model was considered using the methods of RG and OPE in [18]. It was shown that the main contribution to the OPE of the S_{2n} gives a family of scalar operators of the form $(\partial\varphi)^{2n}$. Renormalization of the operators is not multiplicative, but includes operator mixing. The mixing complicates greatly the calculation of the anomalous exponents of the operators; obtaining an analytic expression of the anomalous exponents for arbitrary n does not succeed even in the one-loop approximation. In [18], the one-loop approximation was considered for the family of the S_4; the exponents were evaluated in the linear on ε approximation, the negative exponent were found among them, so the existence of the anomalous scaling was proved in the model of the vector advection with the mixing of the operators. It was in the case of flat flows ($d = 2$) [21] that the anomalous exponents of the structure functions of the arbitrary order were calculated in the model (4), (7), (8). The anomalous exponents were obtained in the one-loop approximation coincide with the exponents (5) of the scalar admixture. 'The most dangerous' operators were found to be the powers of dissipation operator, just as in the scalar model.

4. The bases for scalar operators of the form $(\partial\varphi)^n$

As demonstrated in [18], the leading terms in (2) arise from the family $\Phi^{(n)}$ of scalar operators of the form $(\partial\varphi)^n$. More precisely, that family consists of all possible contractions of n tensors $\partial_i \varphi_j$. Denoting the symmetric and the antisymmetric parts of the $\partial_i \varphi_j$ as

$$S_{ij} \equiv (\partial_i \varphi_j + \partial_j \varphi_i)/2, \qquad A_{ij} \equiv (\partial_i \varphi_j - \partial_j \varphi_i)/2, \tag{9}$$

we can express for S_4 the seven operators which form the family $\Phi^{(4)} = \{\operatorname{tr}(A^2)^2, \operatorname{tr}(S^2)^2,$ $\operatorname{tr}(A^2)\operatorname{tr}(S^2), \operatorname{tr}(A^2 S^2), \operatorname{tr}((AS)^2), \operatorname{tr}(A^4), \operatorname{tr}(S^4)\}$. The main difficulty in the calculation of the critical dimensions of the operators from $\Phi^{(n)}$ is the rapid growth of the $\Phi^{(n)}$ cardinality with n. For example, the family $\Phi^{(6)}$ contains 24 operators, $\Phi^{(8)}$—81, $\Phi^{(10)}$—278 and $\Phi^{(18)}$—as many as 47 246 operators. Fortunately, when space dimension d is sufficiently small not all of the operators are independent. For the most interesting cases $d = 2$ and $d = 3$, the elimination of redundant operators reduces drastically the size of the matrix of critical dimensions.

Our goal is to construct for $d = 2$ and $d = 3$ the sets Ω_d of minimal size which contains invariants of the matrix $\partial_i \varphi_j$. Then we will form bases of $\Phi^{(n)}$ using products of the invariants from Ω_d.

4.1. 2D case

The dependencies mentioned above can be considered as consequences of the Hamilton–Cayley identity. For a traceless 2×2 matrix M it has the form:

$$\chi_2(M) = M^2 - I \operatorname{tr}(M^2)/2 = 0, \tag{10}$$

with the identity matrix I. From (10) one has $\chi_2(\alpha A + \beta S) = 0$ for any linear combination of the matrices A and S from (9). Collecting the coefficients of three independent structures α^2, β^2 and $\alpha\beta$ gives the three relations:

$$A^2 - I\operatorname{tr}(A^2)/2 = 0, \qquad S^2 - I\operatorname{tr}(S^2)/2 = 0, \qquad AS + SA = 0. \tag{11}$$

The first and second relations are the Hamilton–Cayley identities for the matrices A and S. The last one can be interpreted as a commutation relation. The relations (11) can be written in the form

$$A^2 \sim 0, \qquad S^2 \sim 0, \qquad SA \sim -AS, \tag{12}$$

where $\sim$ means the equality up to polynomials of lesser degree.

It follows from (12) that $P_3(A, S) \sim 0$ for any polynomial of the third degree, so that an arbitrary polynomial of the matrices A and S can be written as $P(A, S) = C_1(\Omega_2)AS + C_2(\Omega_2)A + C_3(\Omega_2)S + C_4(\Omega_2)I$, where the coefficients are polynomials of the scalar invariants from the minimal set

$$\Omega_2(A, S) = \{\operatorname{tr}(A^2), \operatorname{tr}(S^2)\} \tag{13}$$

and, as a consequence, $\operatorname{tr}(P(A, S)) = 2C_4(\Omega_2)$.

Thus for $d = 2$ each operator from $\Phi^{(2n)}$ is a polynomial $P_n(\operatorname{tr}(A^2), \operatorname{tr}(S^2))$, which can be decomposed into the basis

$$\operatorname{tr}(A^2)^{n-k}\operatorname{tr}(S^2)^k, \qquad 0 \leqslant k \leqslant n. \tag{14}$$

4.2. 3D case

The Hamilton–Cayley identity for a traceless 3×3 matrix M has the form:

$$\chi_3(M) = M^3 - M\operatorname{tr}(M^2)/2 - I\operatorname{tr}(M^3)/3 = 0. \tag{15}$$

From the Hamilton–Cayley identity for the matrices A and S from (9) it follows that the minimal set $\Omega_3(A, S)$ contains the invariants $\operatorname{tr}(A^2)$, $\operatorname{tr}(S^2)$ and $\operatorname{tr}(S^3)$. Equating to zero the

coefficients of $\alpha^2\beta$ and $\alpha\beta^2$ in the identity $\chi_3(\alpha A + \beta S) = 0$ gives

$$A^2 S + ASA + SA^2 - S\operatorname{tr}(A^2)/2 - I\operatorname{tr}(A^2 S) = 0. \tag{16}$$

$$AS^2 + SAS + S^2 A - A\operatorname{tr}(S^2)/2 = 0. \tag{17}$$

It follows from (16) that the invariant $\operatorname{tr}(A^2 S)$ is in the minimal set $\Omega_3(A, S)$. Neglecting the polynomials of lesser degrees in (15)–(17) results in commutation-like relations

$$A^3 \sim 0, \qquad S^3 \sim 0, \tag{18}$$

$$SAA \sim -AAS - ASA, \qquad SSA \sim -ASS - SAS. \tag{19}$$

It follows from (18) that each monomial containing A^3 or S^3 vanishes. Each monomial containing SAA or SSA can be decomposed according to (19) as a sum of other monomials each of which precedes the initial monomial in the lexicographic ordering. It then follows that the iterations of (18) and (19) finally give an expression without any one of the factors A^3, S^3, SA^2 and $S^2 A$. Applying (18) and (19) to a polynomial of the third degree gives

$$P_3(A, S) \sim L(A^2 S, ASA, AS^2, SAS), \tag{20}$$

where L is a linear combination of the arguments. Moreover, from relation (20) it follows that $\operatorname{tr}(P_3(A, S)) = L(\operatorname{tr}(A^2), \operatorname{tr}(A^2 S), \operatorname{tr}(S^2), \operatorname{tr}(S^3))$.

Applying (18) and (19) to a polynomial of the fourth degree gives

$$P_4(A, S) \sim L(A^2 SA, A^2 S^2, ASAS, SASA, SAS^2), \tag{21}$$

but the expression allows further simplification. The reason is the existence of another relations besides (15)–(17). For example, none of the relations (18), (19) can be applied to the monomial $ASASAS$, but from the Hamilton–Cayley identity for the matrix AS it follows that $(AS)^3 \sim 0$. Applying (15)–(17) to the coefficient of $\alpha\beta\gamma$ in the identity $\chi_3(\alpha A + \beta S + \gamma AS) = 0$ gives the equation

$$A^2 S^2 - (SA)^2 - A^2 \operatorname{tr}(S^2)/2 - S^2 \operatorname{tr}(A^2)/2 + I\operatorname{tr}(A^2 S^2) + I\operatorname{tr}((AS)^2) = 0. \tag{22}$$

Neglecting the polynomials of lesser degrees results in

$$SASA \sim A^2 S^2 \tag{23}$$

that turns (21) into $P_4(A, S) \sim L(A^2 SA, A^2 S^2, ASAS, SAS^2)$. Taking the trace of (22) gives the equation that relates $\operatorname{tr}(A^2 S^2)$ and $\operatorname{tr}((AS)^2)$:

$$4\operatorname{tr}(A^2 S^2) + 2\operatorname{tr}((AS)^2) - \operatorname{tr}(A^2)\operatorname{tr}(S^2) = 0. \tag{24}$$

Thus we have found all possible relations for the fourth degree monomials. From (22) and (24) the necessity follows to add one more invariant to $\Omega_3(A, S)$ — $\operatorname{tr}(A^2 S^2)$, $\operatorname{tr}((AS)^2)$ or their linear combination independent on (24), for example, the invariant $\operatorname{tr}([A, S]^2)$ with the commutator $[A, S] \equiv AS - SA$.

For the polynomials of the fifth degree, applying (18), (19) and (23) gives $P_5(A, S) \sim L(A^2 SAS, ASAS^2)$. To find relations for the fifth degree monomials we equate to zero the coefficient of $\alpha\beta^2$ in the identity $\chi_3(\alpha S + \beta AS) = 0$ after simplification that results in $I\operatorname{tr}(ASAS^2) + I\operatorname{tr}(A^2)\operatorname{tr}(S^3)/6 = 0$. Consequently, there are no new invariants to add to the $\Omega_3(A, S)$.

Using (18), (19) and (23) for the polynomials of the sixth degree gives the relation $P_6(A, S) \sim L(A^2 SAS^2)$, specifically, it follows $[A, S]^3 \sim -6A^2 SAS^2$. Analysing the

identity $\chi_3([A, S]) = 0$ we conclude that $[A, S]^3 \sim 0$. Another invariant $\text{tr}([A, S]^3) = -6\,\text{tr}(A^2 S A S^2)$ is added to the set Ω_3.

Finally, $P_6(A, S) \sim 0$, and hence any polynomial of the matrices A and S can be represented as a polynomial of the degree not higher than five with coefficients which are polynomials of $\Omega_3(A, S)$. For the trace of each polynomial we have $\text{tr}(P(A, S)) = P(\Omega_3)$. The complete minimal set of invariants of the traceless 3×3 matrix (9) is

$$\Omega_3(A, S) = \{\text{tr}(A^2), \text{tr}(S^2), \text{tr}(A^2 S), \text{tr}(S^3), \text{tr}([A, S]^2), \text{tr}([A, S]^3)\}. \tag{25}$$

To construct the basis of the family $\Phi^{(n)}$ we need to take into account additional relation among invariants in (25). Namely, that the last invariant is similar to pseudoscalar and its square is expressed as the polynomial on the others invariants. To prove that we rewrite (15) in the form $I\,\text{tr}(M^3)/3 = M^3 - M\,\text{tr}(M^2)/2$ substituting $M = [A, S]$. After squaring and applying of (16), (17) and (22) to the right-hand side we obtain the desired relation. Thus for $d = 3$ any operator from $\Phi^{(n)}$ can be decomposed into the basis

$$\begin{aligned}
&\text{tr}(A^2)^{n_1}\,\text{tr}(S^2)^{n_2}\,\text{tr}(A^2 S)^{n_3}\,\text{tr}(S^3)^{n_4}\,\text{tr}([A, S]^2)^{n_5}\,\text{tr}([A, S]^3)^{n_6}, \\
&n = 2n_1 + 2n_2 + 3n_3 + 3n_4 + 4n_5 + 6n_6, \qquad n_6 \leqslant 1.
\end{aligned} \tag{26}$$

5. Conclusion

The leading contribution to the inertial-range behaviour of the structure functions of the vector admixture φ has the form $S_n(r) \sim r^{n(1-\varepsilon/2)}(r/L)^{-\xi_n}$, where the anomalous exponents ξ_n are given by eigenvalues of the matrix of critical dimensions of the family $\Phi^{(n)}$ of scalar operators of the form $(\partial\varphi)^n$, see [18]. The main results of the paper are finding the minimal sets Ω_d of the invariants of the tensor $\partial_i \varphi_j$ for the spatial dimensions $d = 2$ (13) and $d = 3$ (25). The method proposed for invariants searching is based on the Hamilton–Cayley identity and can be generalized to searching of the joint invariants of the several tensors for arbitrary spatial dimension. Using the invariants from Ω_d the bases of the family $\Phi^{(n)}$ were constructed for arbitrary n in two dimensions (14) and three dimensions (26).

For $d = 2$, the basis (14) has clear physical sense as $\text{tr}(S^2) \propto \Phi_{\text{dis}}$—the dissipation operator and $\text{tr}(A^2) \propto |\text{rot}\,\varphi|^2$—the enstrophy operator. The matrix of the critical dimensions become triangular in the basis. Just as in the scalar admixture case the inertial range behaviour of the structure function is determined by the power of the dissipation operator. So the basis (14) provides the possibility of solving problems in general. With another representation of the basis (14) it was obtained $\xi_n = \varepsilon n(n-2)/8 + O(\varepsilon^2)$ in [21] that coincides with the exponents (5) of the scalar case.

For $d = 3$, the matrix of critical dimensions in the basis (26) has a general form. However, taking into account the dependencies between the operators radically reduces the size of the matrix. For example, the family $\Phi^{(18)}$ contains 47 246 operators for general d, but only 154 of them appear independent for $d = 3$. This gives the possibility of calculating the anomalous exponents (in the linear approximation on ε): $\xi_4 \approx 0.546\varepsilon$, $\xi_6 \approx 1.75\varepsilon$, $\xi_8 \approx 3.66\varepsilon$, $\xi_{10} \approx 6.27\varepsilon$, $\xi_{12} \approx 9.58\varepsilon$, $\xi_{14} \approx 13.6\varepsilon$, $\xi_{16} \approx 18.3\varepsilon$, $\xi_{18} \approx 23.7\varepsilon$.

Acknowledgments

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